ANALYSIS OF TERMINAL CAPACITIES IN LOSSY COMMUNICATION NETS

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THESIS

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IN LOSSY COMMUNICATION NETS

by

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June 1973

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Analysis of Terminal Capacities
in Lossy Communication Nets

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1973

ABSTRACT

Properties of lossy communication nets are studied in this paper. Optimum flows in general and special lossy nets are discussed. Methods for the calculations of terminal capacities in the nets are presented.

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ACKNOWLEDGEMENT

The author offers his gratitude and appreciation to Professor Shu-Gar Chan for his advice and assistance during the preparation of this thesis.

I. LOSSY COMMUNICATION NETS

A communication net is a system composed of stations and interconnecting transmission channels. It can be conveniently represented by a linear graph. The analysis of communication nets often employ the techniques developed in graph theory. In this paper certain selected terms will be defined. However, the definitions of basic terminology will not be repeated here as they can be found in standard textbooks in graph theory [5].

Communication flows in a net have been investigated by many authors [1]-[10]. In the conventional model of the net, the flow through the net in a channel is limited by its channel capacity. The flow is conserved at the stations and suffers no loss during transmission. There are many flow problems for which the conventional model is inadequate. For example, in many practical nets, the flow suffers losses during transmission due to leakages and damages.

This paper deals with communication nets with losses. These nets are called "lossy communication nets," or in short, "lossy nets," which are described by Mayeda [5] as follows:

A model of a communication net can be represented by an associated oriented graph in which two weights, capacity and efficiency, are given to each edge. Thus, if flow Φ enters an edge which is less than or equal to the capacity of the edge and whose orientation agrees with the orientation of the edge, then the flow that leaves the edge is $\alpha\Phi$ where α is the efficiency, and the loss of flow is $(1-\alpha)\Phi$. An example of flows in a lossy net is illustrated in Fig. 1.

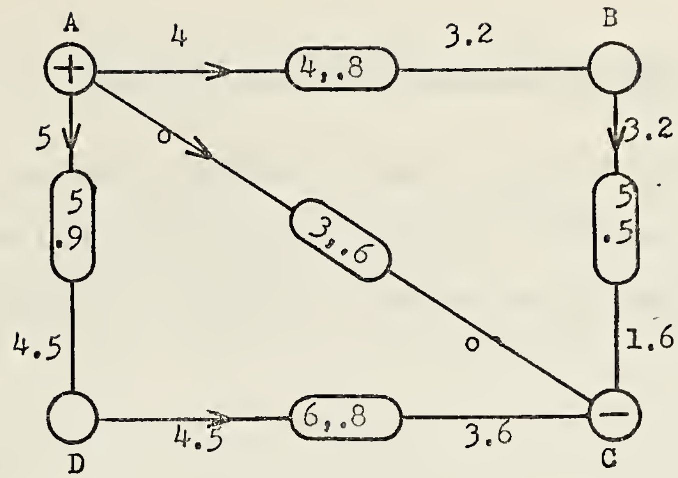

Fig. 1. A lossy communication net

This net is a directed connected graph. There are two specific nodes in this net: node A with plus sign is the source and node C with minus sign is the sink [3]. Associated with each edge are two positive numbers. They are the capacity and the efficiency of the edge. For example, in Fig. 1 the capacity of edge AB is 4 and the efficiency is 0.8. The outgoing flow from node A in branch $\overrightarrow{AB}$, where the arrow indicates the direction of the branch or edge, cannot exceed the capacity 4 and is actually 4 in Fig. 1. The incoming flow to node B is 3.2 which is the efficiency multiplied by the outgoing flow from node A.

Notice that the conservation law must hold at every node except for the source and the sink. For instance, at the intermediate node D the total outgoing flow and the total incoming flow are both equal to 4.5. Since the flow from node A to node B is equal to the capacity, edge AB is said to be saturated. The flow from node A to node C along the edge AC is equal to zero, thus edge AC is said to be void.

II. PROPERTIES OF LOSSY COMMUNICATION NETS

In this section properties of lossy nets will be discussed. A number of definitions will first be given. This is followed by several theorems which may be applied in the analysis of the various types of nets.

A. SEMICUTS

Consider the cutset $S = (g,h,i)$ in the oriented net G shown in Fig. 2.

Definition 1. Let g_1 and g_2 be the separated subnets obtained from G by deleting all edges in S, then g_1 and g_2 are called the corresponding subnets of S. Let vertex i be in g_1 and vertex j in g_2, then the cutset is said to separate verties i and j. The symbol S_{ij} is used to represent such a cutset.

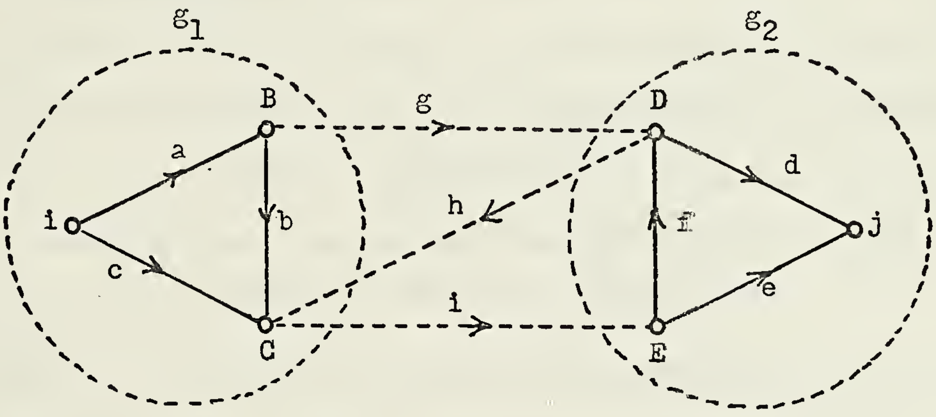

Fig. 2. Illustration for Definition 1

Definition 2. Let S_{ij} be a cutset and g_1 and g_2 be the corresponding subnets of S where vertex $i \in g_1$ and vertex $j \in g_2$, then s_{ij} is a subset of S_{ij} such that

$$s_{ij} = \left\{ b(U,V) \mid b \in S_{ij},\ U \in g_1,\ V \in g_2 \right\}$$

where $b(U,V)$ is an edge connected from vertex U to vertex V. Similarly, s_{ji} is a subnet of S_{ij} such that

$$s_{ji} = \left\{ b(U,V) \mid b \in S_{ij},\ V \in g_1,\ U \in g_2 \right\}$$

s_{ij} and s_{ji} are called semicuts of S_{ij}.

Notice that $s_{ij} \cup s_{ji} = S_{ij}$ and $s_{ij} \cap s_{ji} = 0$

For example, in Fig. 2, $s_{ij} = (g,i)$ and $s_{ji} = (h)$

Definition 3. $V[s(c)]$ indicates the values of semicuts defined as $V[s(c)] = \sum_{b_q \in s} C_q$, where C_q is the capacity of edges b_q.

We next discuss the method of assigning flow Φ to a net. Let $P = \left\{ b_1, b_2, \ldots, b_k \right\}$ be a directed path in a lossy net from vertex i to vertex j. We assign flow $\Phi_{ij}^{(p)}$ to b_p ($p=1,2,\ldots,k$) as follow:

$$\Phi(b_1) = \Phi_{ij}^{(p)},\ \Phi(b_2) = \alpha_1 \Phi(b_1),\ \ldots\ldots,\ \Phi(b_k) = \alpha_{k-1} \Phi(b_{k-1}),$$

Under the condition that

$$\Phi(b_p) \leq C_p - \Phi_o(b_p) \quad \text{for} \quad p=1,2,\ldots,k$$

where C_p is the capacity of edge b_p, α_p is the efficiency of edge b_p, and $\Phi_o(b_p)$ is the flow which had been assigned to edge b_p previously.

Let $P_1, P_2, \ldots, P_k$ be all the possible directed paths from vertex i to vertex j in net G, then we can assign flow $\Phi_{ij}^{(P_1)}, \Phi_{ij}^{(P_2)}, \ldots, \Phi_{ij}^{(P_k)}$ to $P_1, P_2, \ldots, P_k$ respectively, such that the total flow $\Phi_{ij} = \sum_{p=1}^{k} \Phi_{ij}^{(P_p)}$. It is clear that any flow Φ_{ij} assigned to net G can be broken into flows $\Phi_{ij}^{(P_1)}, \Phi_{ij}^{(P_2)}, \ldots, \Phi_{ij}^{(P_k)}$ which are assigned to $P_1, P_2, \ldots, P_k$ respectively.

For convenience, we say that assigning flow Φ_{ij} to net G means that we are assigning flow $\Phi_{ij}^{(P_p)}$, where $\sum_{p=1}^{k} \Phi_{ij}^{(P_p)} = \Phi_{ij}$, to directed paths

$P_1, P_2, \ldots, P_k$ which are from vertex i to vertex j in net G.

Depending on the values of $\Phi_{ij}^{(P_1)}, \Phi_{ij}^{(P_2)}, \ldots, \Phi_{ij}^{(P_k)}$ the total flow $\Phi_{ij} = \sum_{p=1}^{k} \Phi_{ij}^{(P_p)}$ will be different. However, for any given G which consists of finite number of edges with finite capacity for each edge, there will exist a largest total flow, Φ_{ij}, which is finite.

Example 1. Consider the lossy communication net G of Fig. 3a. The first number in the parentheses of each edge is the capacity, the second is the efficiency of the edge, and the last is the total flow Φ_o entering the edge from the previous assignment of flows. Hence, initially $\Phi_o = 0$ for all edges. One way of assigning the flow, $\Phi_{ij} = 10$, from i to j in the net G is as follow: First, we assign a flow $\Phi_{ij}^{(P_1)} = \Phi(a) = 4$ to the path $P_1 = (a,b,c)$, the flow $\Phi(b)$ through the edge b is equal to $\alpha_a \Phi(a) = .5 \times 4 = 2$, and the flow $\Phi(c)$ through the edge c is equal to $\alpha_b \Phi(b) = .8 \times 2 = 1.6$. All the values of $\Phi(a)$, $\Phi(b)$, and $\Phi(c)$ are shown in Fig. 3b. Notice that $\Phi_o(a)$ of edge a becomes 4, $\Phi_o(b)$ of edge b becomes 2 and $\Phi_o(c)$ of edge c becomes 1.6. Now we assign a second flow $\Phi_{ij}^{(P_2)} = \Phi(a) = 6$ to the path $P_2 = (a,d,e)$, the flow $\Phi(d)$ through the edge d is equal to $\alpha_a \Phi(a) = .5 \times 6 = 3$, and the flow $\Phi(e)$ through the edge e is equal to $\alpha_d \Phi(d) = .7 \times 3 = 2.1$ as shown in Fig. 3c. The total flow Φ_{ij} from node i to node j becomes $\Phi_{ij}^{(P_1)} + \Phi_{ij}^{(P_2)} = 4 + 6 = 10$.

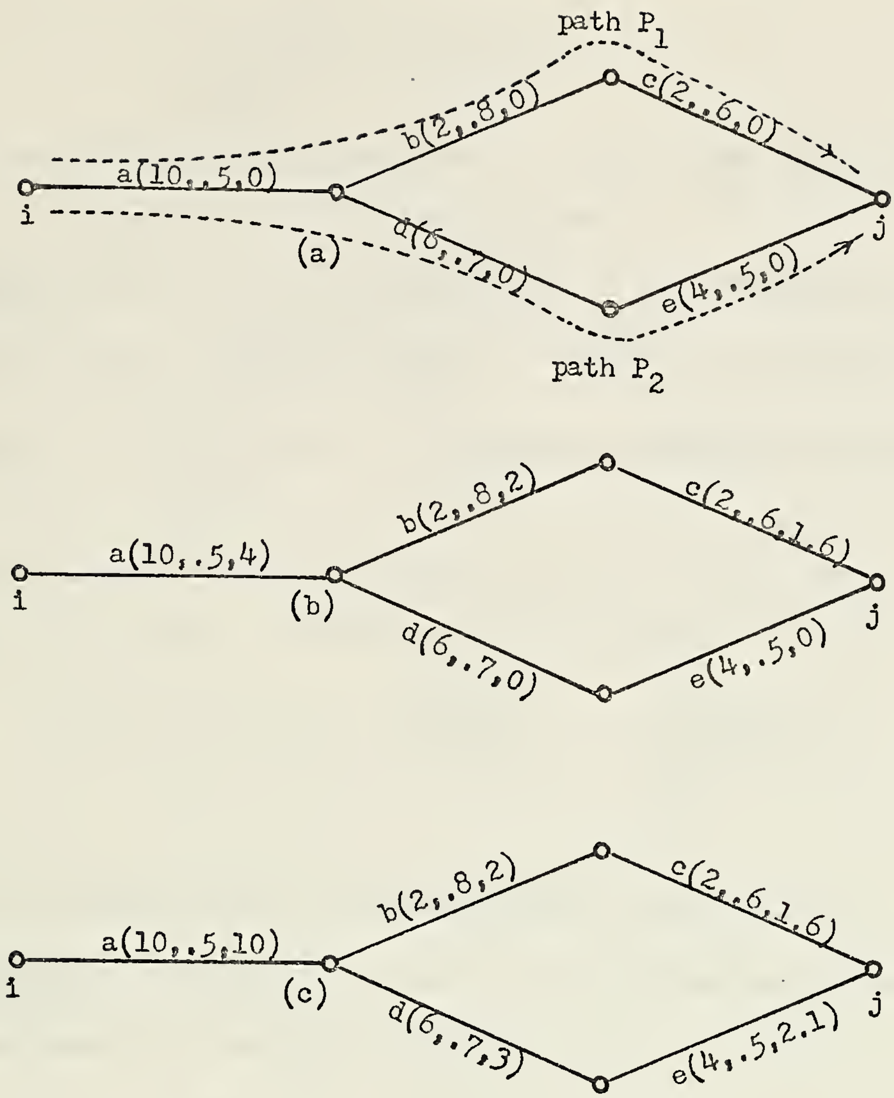

Fig. 3. Illustration for Example 1

B. BASIC SATURATED CUTSETS

__Definition 4.__ An edge i is said to be saturated if the flow $\Phi(i)$ assigned to the edge is equal to the capacity of the edge.

For example, in the Fig. 3c, edge a is saturated, and edge d is not saturated.

__THEOREM 1.__ Let G be a lossy communication net. Suppose that flow Φ_{ij} from vertex i to vertex j has been assigned to G.

If there is no cut set S_{ij} such that

$$\sum_{b_r \in s_{ij}} \Phi(b_r) = V[\, s_{ij}(c)]$$

then there exists a direct path from i to j such that an additional flow $\Phi > 0$ from i to j can be assigned.

<u>PROOF:</u> Let G' be a net obtained from G by deleting every saturated edge. Since there exists no cut set S_{ij} such that

$\sum_{b_r \in s_{ij}} \Phi(b_r) = V[s_{ij}(c)]$, G' contains a subnet in which there exists at least one direct path $P = (b_1, b_2, \ldots, b_k)$ from i to j. Hence, we can assign nonzero flow Φ to G where

$$\Phi \leq \min \left\{ c_1 - \Phi_o(b_1),\ \frac{c_2 - \Phi_o(b_2)}{\alpha_1}, \ldots,\ \frac{c_k - \Phi_o(b_k)}{\prod_{m-1}^{k-1} \alpha_m} \right\}$$

$\Phi_o(b_p)$ is the flow which has been assigned to edge b_p in G.

<u>Definition 5.</u> Cut set S_{ij} is said to be a saturated cut set under the assignment of flow Φ_{ij} to G if

$$\sum_{b_r \in s_{ij}} \Phi(b_r) = V[\, s_{ij}(c)\,]$$

Notice that if there exists no saturated cut set S_{ij} under the assignment of a flow Φ_{ij} to G, it is then possible to assign additional flow Φ'_{ij} to G.

<u>Definition 6.</u> Cut set S_{ij} is said to be a basic saturated cut set under the assignment of a flow Φ_{ij} to G if S_{ij} is a saturated cut set and satisfies $\sum_{b_t \in s_{ji}} \Phi(b_t) = 0$

where s_{ij} and s_{ji} are the pair of semicuts of S_{ij}.

<u>Example 2.</u> Consider a lossy communication net G in Fig. 4a. Let P_1, P_2, and P_3 be directed paths from i to j. If we assign $\Phi_{ij} = 10$ to P_2 as shown in Fig. 4b, we have saturated cutset which

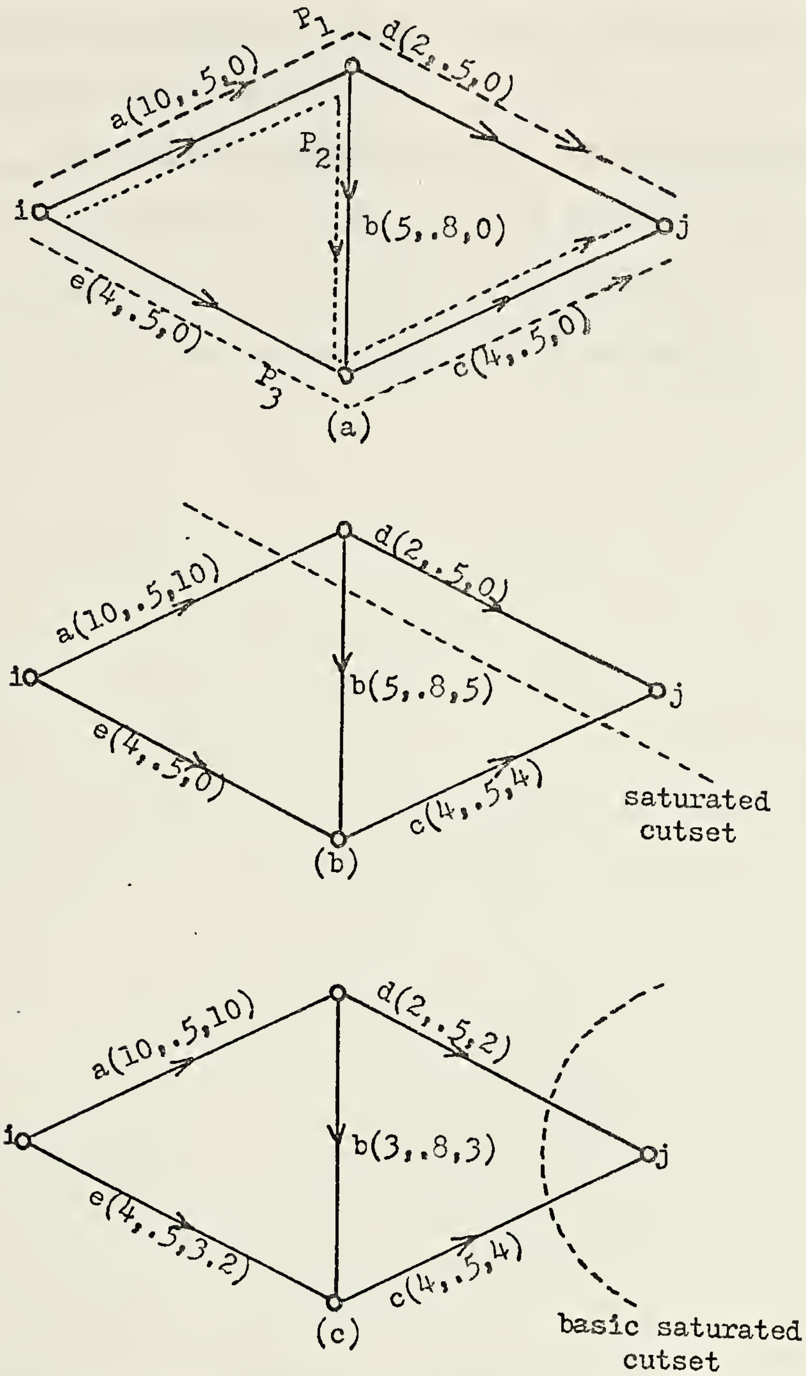

Fig. 4. Illustration for Example 2

consists of saturated edges a, b, and c. However, this cut set is not a basic saturated cut set because $\Phi(b) \neq 0$. By assigning $\Phi_{ij}^{(P_1)} = 4$ to P_1, $\Phi_{ij}^{(P_2)} = 6$ to P_2, and $\Phi_{ij}^{(P_3)} = 3.2$ to P_3 we have the result shown in Fig. 4c. This example shows that we have a saturated cut set consisting of edges c and d which also constitute a basic saturated cut set.

THEOREM 2. For a lossy communication net G, there exists at least one flow Φ_{ij} assigned to G such that there exists at least one basic saturated cut set S_{ij} in G with this flow.

The proof of this theorem has been given by Mayeda [5].

III. OPTIMUM FLOWS IN A COMMUNICATION NET

A. GENERAL CASES

Definition 7. A maximum flow Φ_{ij} from i to j which can be assigned to a net G in order to receive the maximum flow at j is called a source terminal capacity symbolized by $\overline{t}_{ij}$. The maximum flow which will be received at vertex j when Φ_{ij} is assigned to G is called sink terminal capacity symbolized by $\underline{t}_{ij}$.

The following two theorems are taken from reference [6].

THEOREM 3. If and only if there exists a basic saturated cut set which separates i and j, the flow Φ_{ij} assigned to an oriented net is maximum.

THEOREM 4. Terminal capacity $\overline{t}_{ij}$ is equal to

$$\overline{t}_{ij} = \min \left\{ V \left[S_{pij} \right] \right\};$$

semicut s_{pij} of $S_{pij} \in \left\{ S_{ij} \right\}$.

Example 3. From the oriented net G in Fig. 5, S_{ij} consists of the following cut sets which separate i and j:

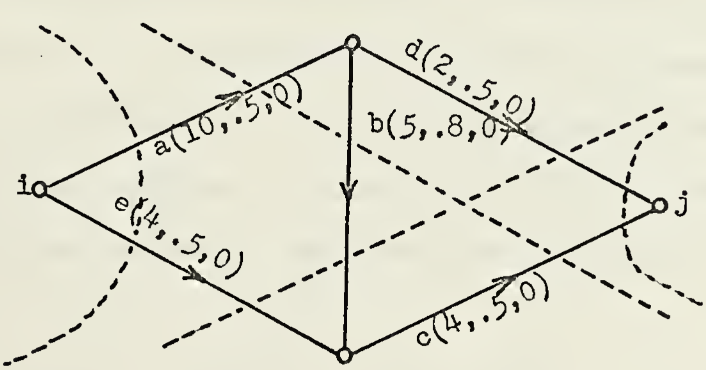

Fig. 5. Cut sets for Example 3

$S_{1ij} = (a,e)$ $S_{2ij} = (a,b,c)$ $S_{3ij} = (d,c)$ $S_{4ij} = (e,b,d)$

The semicuts s_{pij} of these cut sets are:

$s_{1ij} = (a,e)$ $s_{2ij} = (a,c)$ $s_{3ij} = (d,c)$ $s_{4ij} = (e,b,d)$

and by definition 3

$$V\,(s_{1ij}) = 10 + 4 = 14$$
$$V\,(s_{2ij}) = 10 + 4 = 14$$
$$V\,(s_{3ij}) = 2 + 4 = 6$$
$$V\,(s_{4ij}) = 4+5+2 = 11$$

$$\bar{t}_{ij} = \min \left\{ V\,(s_{pij}) \right\} = \min \left\{ 14,14,6,11 \right\} = 6$$

<u>Definition 8.</u> For a fixed receiving flow value, a flow Φ is said to be optimum if its sending flow value is minimum.

If a flow Φ_o with the sending flow value $\bar{t}$ and the receiving flow value $\underline{t}$ is already assigned to the net N and if an increment Δ with sending flow value $\epsilon > 0$ is assigned to a flow return sequence q, then the resultant flow $\Phi_1 = \Phi_o + \Delta$ has the same receiving flow value $\underline{t}$ as flow Φ_o and that the sending flow value of Φ_1 is increased by the amount $\epsilon - \epsilon'$, where ϵ' is the amount of flow returned to the source.

Let F be the set of forward edges in the flow return sequence, and let B be the set of backward edges in the flow sequence. Then ϵ' is expressed as the product of ϵ and the efficiency factor product α_q of the flow return sequence q, that is $\epsilon' = \epsilon \times \alpha_q$ where the efficiency factor product of the flow return sequence is

$$\alpha_q = \prod_{e_m \in F} \alpha_{e_m} \times \prod_{e_n \in B} \frac{1}{\alpha_{e_n}} .$$

Note that if the efficiency factor product α_q of a flow return sequence q is greater than unity, then the resultant sending flow value

$$\bar{t}' = \bar{t} + \epsilon - \epsilon' = \bar{t} + (1-\alpha_q)\,\epsilon$$

is less than $\bar{t}$.

Notice that to have the same receiving flow value $\underline{t}$, it is possible that the sending flow value $\bar{t}'$ can be less than $\bar{t}$. In this case the flow is not optimum. This leads to the next theorem.

THEOREM 5. A flow is optimum if and only if there is no flow return sequence with respect to the source whose efficiency factor product is greater than unity.

Note that if the net N has uniform edge efficiency, $\alpha_{em} = \alpha_{en} = \alpha < 1$ for any m and n, then

$$\alpha_q = \alpha^m \times \frac{1}{\alpha^n} = \alpha^{m-n}$$

where m is the number of forward edges and n is the number of backward edges. If $n > m$ applies $\alpha_q > 1$.

Example 4. Consider the net N with uniform edge capacity as shown in Fig. 6.

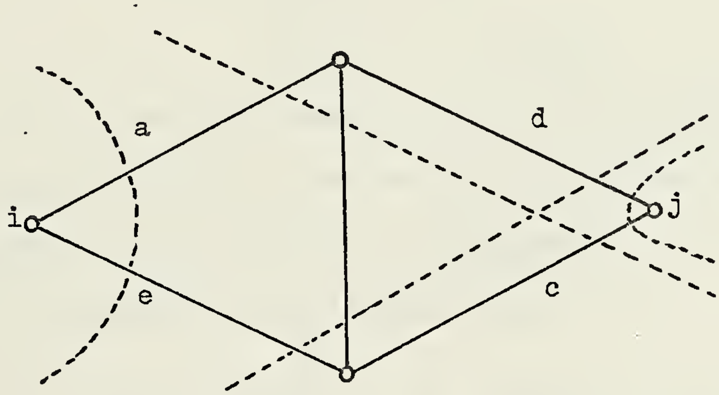

Fig. 6. Net with uniform edge capacity

If all the edge capacities are equal to 10, then the semicuts s_{pij} are $s_{1ij} = (a,e)$, $s_{2ij} = (a,c)$, $s_{3ij} = (d,c)$, $s_{4ij} = (e,b,d)$, and

$$V(s_{1ij}) = 10 + 10 = 20$$

$$V(s_{2ij}) = 10 + 10 = 20$$

$$V(s_{3ij}) = 10 + 10 = 20$$

$$V(s_{4ij}) = 10+10+10 = 30$$

$$\bar{t}_{ij} = \min \left\{ V[\, s_{pij} \,] \right\} = \min (20,20,20,30) = 20$$

In this example the semicut s_{1ij} has only two edges, if n is the smallest number of edges of the semicut s_{pij}, then the source terminal capacity $\bar{t}_{ij}$ is equal to n times the edge capacity.

B. SPECIAL CASES.

Consider a flow Φ from a node called source i to another node called sink j. The flow in the communication net N is a non-negative real function of two variables, edge and node, such that the following conditions are satisfied:

(a) If e is an edge connected from node x to node y, then:

$$0 \leq \Phi(e,x) \leq C_e$$

$$\Phi(e,y) = \alpha_e \Phi(e,x)$$

where C_e is the capacity of edge e; $\alpha_e \leq 1$ is the efficiency of edge e; $\Phi(e,x)$ and $\Phi(e,y)$ are the amounts of flow passing through edge e at node x and at node y respectively.

(b) Flow is conserved at the internal nodes

$$\sum_{e_i \in A_x} \Phi(e_i,x) - \sum_{e_j \in B_x} \Phi(e_j,x) = 0$$

Where A_x is the set of edges issuing from node x and B_x is the set of edges entering into node x.

(c) The sending and receiving flow value $\overline{t}$ and $\underline{t}$, respectively are non-negative;

$$\overline{t} = \sum_{e_n \in A_i} \Phi(e_n, i) - \sum_{e_m \in B_i} \Phi(e_m, i) \geq 0$$

$$\underline{t} = \sum_{e_m \in B_j} \Phi(e_m, j) - \sum_{e_n \in A_j} \Phi(e_n, j) \geq 0$$

where nodes i and j are the source and the sink, respectively.

1. <u>Flow in one path</u>

Let all edges have the same capacity C. We obtain the following theorem.

<u>THEOREM 6.</u> If there is only one path connected from node i to node j, then the maximum receiving flow $\underline{t}_{ij}$ at node j is equal to

$$C \prod_{i=1}^{n+1} \alpha_i$$

where α_i is the efficiency of edge e_i, and n is the number of internal nodes.

<u>PROOF:</u> By the first part of condition (a), the maximum sending flow $\overline{t}_{ij}$ at node i is C, and by the second part of condition (a), the maximum receiving flow $\underline{t}_{ij}$ at node 1 is $\alpha_1 \overline{t}_{ij} = C\,\alpha_1$. Condition (b), which can be called the conservation condition, must hold at every node except for the source and the sink.

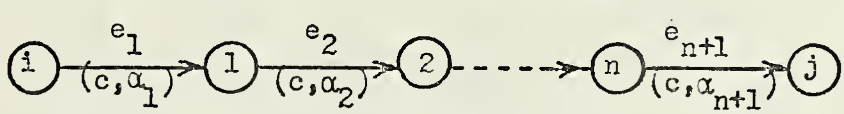

Fig. 7. Illustration for Theorem 6

For instance, at node 1 of Fig. 7, the incoming flow $C\,\alpha_1$ is equal to the outgoing flow. The maximum receiving flow $\underline{t}_{ij}$ at node 2 is $C\,\alpha_1\alpha_2$. The maximum receiving flow $\underline{t}_{ij}$ at node j is equal to

$$C\,\alpha_1\alpha_2\cdots\alpha_{n+1} = C\prod_{i=1}^{n+1}\alpha_i \, .$$

Example 5. In Fig. 8, there is one edge connected from node i to node j, and there is one cut set S_{ij}. The source terminal capacity

$$\bar{t}_{ij} = \min\left\{V\,(s_{ij})\right\} = \min\,(10) = 10.$$

The maximum receiving flow $\underline{t}_{ij}$ at node j is equal to $(10)(.9)=9$ or

$$\underline{t}_{ij} = 90\%\,\bar{t}_{ij} \, .$$

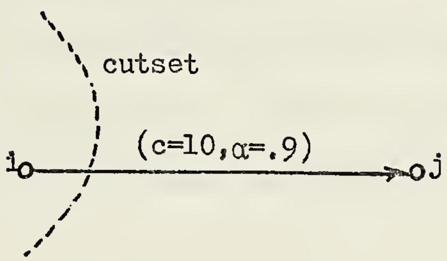

Fig. 8. Net with one edge

Example 6. In Fig. 9, there are two edges connected from i to j. Assume all edges have the same capacity $C=10$ and the same efficiency $\alpha = .9$. In this case,

$$\bar{t}_{ij} = 10 \text{ and } \underline{t}_{ij} = (10)(.9)(.9) = 8.1$$

$$\text{or} \quad \underline{t}_{ij} = 81\%\,\bar{t}_{ij} \, .$$

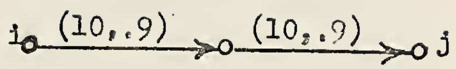

Fig. 9. Net with two edges

Notice that the value of $\underline{t}_{ij}$ decreases when the number of edges increases. If the number of edges is greater than three, then the value of $\underline{t}_{ij}$ is less than 70 percent of $\bar{t}_{ij}$. For example, if the number of edges is 4, then

$$\underline{t}_{ij} = (10)(.9)^4 = 6.561$$

$$\text{or} \quad \underline{t}_{ij} = 65.61\% \ \bar{t}_{ij} .$$

2. Flow in n parallel paths

Again, let all edges have the same capacity C.

THEOREM 7. If there are n paths in parallel connected from node i to node j, then the maximum receiving flow $\underline{t}_{ij}$ at node j is equal to

$$C \sum_{i=1}^{n} \alpha_i$$

PROOF: Refer to Fig. 10, the maximum sending flow for path 1, $(\bar{t}_{ij})_{\text{path 1}}$, at node i is equal to C, and the maximum receiving flow, $(\underline{t}_{ij})_{\text{path 1}}$, at node j is equal to $C\,\alpha_1$. Since there are n paths connected from node i to node j,

$$\bar{t}_{ij} = \min \left\{ V \ (s_{ij}) \right\} = nC$$

$$\text{and} \quad \underline{t}_{ij} = C\,\alpha_1 + C\,\alpha_2 + \ldots\ldots + C\,\alpha_n = C \sum_{i=1}^{n} \alpha_i$$

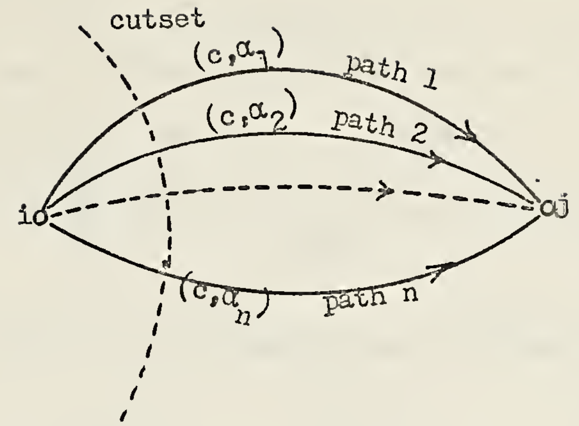

Fig. 10. Illustration for Theorem 7

Example 7. In Fig. 11, let all edges have the same capacity C=10 and efficiency α=.9. In this case $\bar{t}_{ij}$ = (10)(n), where n is the number of edges and $\underline{t}_{ij}$ = (n) (10)(.9) = 9n. Note that for any number of edges, the value of $\underline{t}_{ij}$ is always equal to 90 percent of $\bar{t}_{ij}$; also $\underline{t}_{ij}$ increases if any of the values of n, C, or α increased.

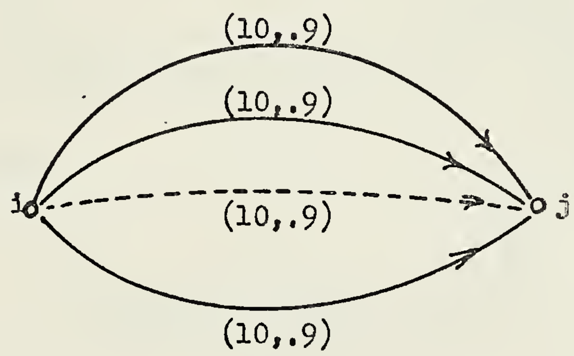

Fig. 11. Net with parallel paths

3. Flow in p paths

THEOREM 8. If there are p paths connected in parallel from node i to node j, and if in each path there are many nodes, then the maximum receiving flow $\underline{t}_{ij}$ at node j is equal to

$$c \prod_{i=1}^{n} \alpha_1^i + c \prod_{j=1}^{m} \alpha_2^j + \dots\dots + c \prod_{k=1}^{1} \alpha_p^1 .$$

PROOF: For path 1 of the net in Fig. 12, Theorem 6 gives

$$(\underline{t}_{ij})_{\text{path 1}} = c \prod_{i=1}^{n} \alpha_i .$$

Since there are p paths in parallel, by Theorem 7,

$$\underline{t}_{ij} = c \prod_{i=1}^{n} \alpha_1^i + c \prod_{j=1}^{m} \alpha_2^j + \dots\dots + c \prod_{k=1}^{1} \alpha_p^1 .$$

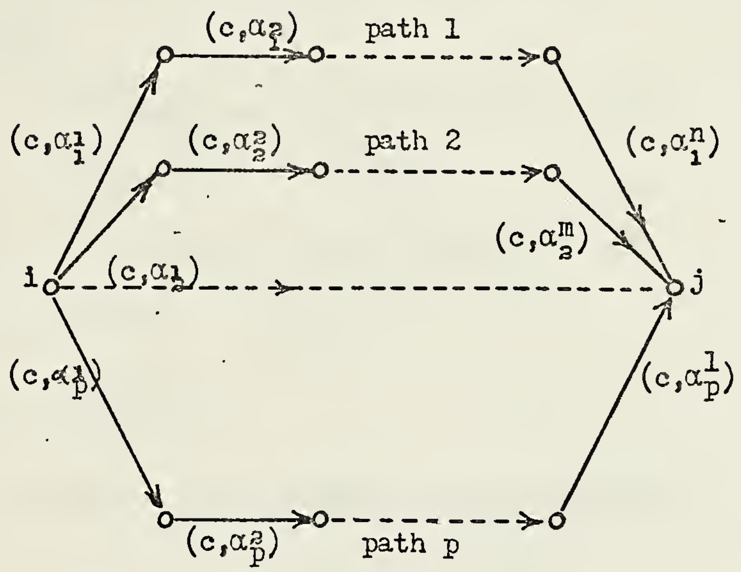

Fig. 12. Illustration for Theorem 8

IV. APPLICATIONS

A. SPECIAL NETS

Example 8. Consider a net as shown in Fig. 13.

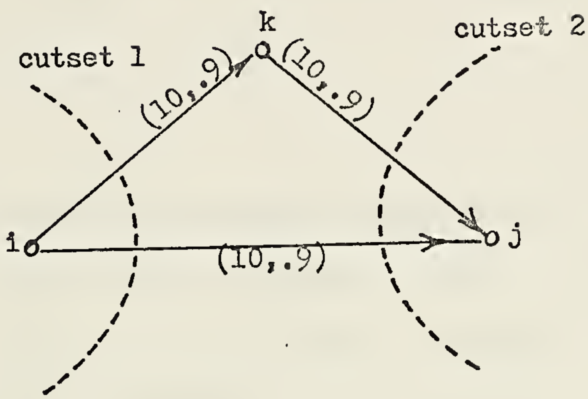

Fig. 13. Illustration for Example 8

In this example,

$$(\underline{t}_{ij})_{\text{path } ijk} = (10)(.9)(.9) = 8.1$$

$$(\underline{t}_{ij})_{\text{path } ij} = (10)(.9) = 9$$

$$\bar{t}_{ij} = \min\left\{V\ (s_{ij})\right\} = \min\ (20,20) = 20$$

$$\underline{t}_{ij} = 9 + 8.1 = 17.1$$

or $\underline{t}_{ij} = 85.5\%\ \bar{t}_{ij}$

Example 9. The net of this example is shown in Fig. 14.

$$(\underline{t}_{ij})_{\text{path } 1} = (10)(.9) = 9$$

$$(\underline{t}_{ij})_{\text{path } 2} = (10)(.9)(.9) = 8.1$$

$$(\underline{t}_{ij})_{\text{path } 3} = (10)(.9)(.9)(.9) = 7.29$$

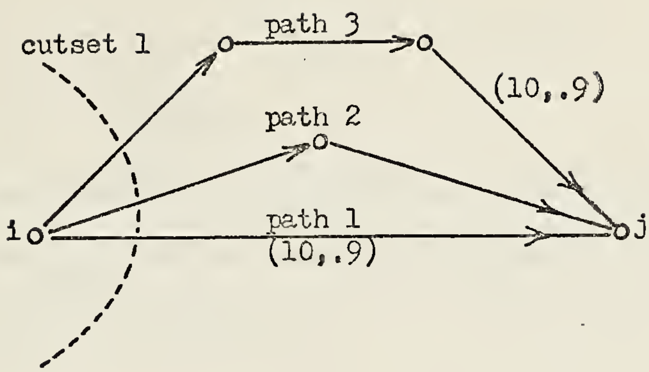

Fig. 14. Illustration for Example 9

$$\overline{t}_{ij} = 3C = (3)(10) = 30$$

$$\underline{t}_{ij} = 9 + 8.1 + 7.29 = 24.39$$

or

$$\underline{t}_{ij} = 82.2\% \; \overline{t}_{ij} .$$

<u>Example 10.</u> The net of this example is shown in Fig. 15. In this example, path 2 has one internal node, path 3 has two internal nodes, and path 4 has three internal nodes.

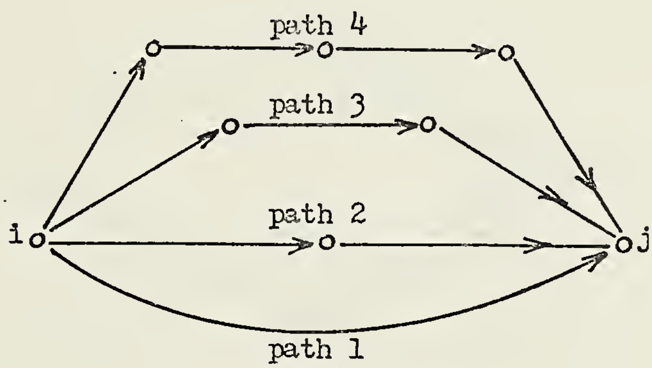

Fig. 15. Illustration for Example 10

$$\bar{t}_{ij} = 40$$

$$\underline{t}_{ij} = 9 + 8.1 + 7.29 + 6.561 = 30.951$$

or $$\underline{t}_{ij} = 76\% \ \bar{t}_{ij} .$$

Notice that the value of $\underline{t}_{ij}$ decreases when the number of paths increases; and $\underline{t}_{ij}$ will be less than 70 percent of $\bar{t}_{ij}$, if the number of paths is greater than 7.

B. SYMMETRIC NETS

Consider a symmetric net with source i and sink j. Suppose that the capacity is large enough to hold many flows at the same time, and all edges have the same capacity. In the net as shown in Fig. 16, assign a flow Φ from node i to node j. Thus,

$$\Phi = \Phi_1 + \Phi_2$$

with the flow Φ_1 following path 1
and the flow Φ_2 following path 2.

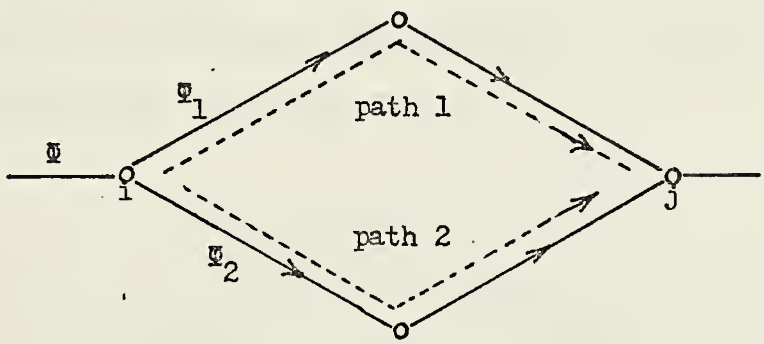

Fig. 16. A symmetric net with two paths

By theorem 8, the total receiving flow at node j is $\alpha^2(\Phi_1 + \Phi_2)$, where α is the capacity.

Now consider the symmetric net with 4 paths as shown in Fig. 17.

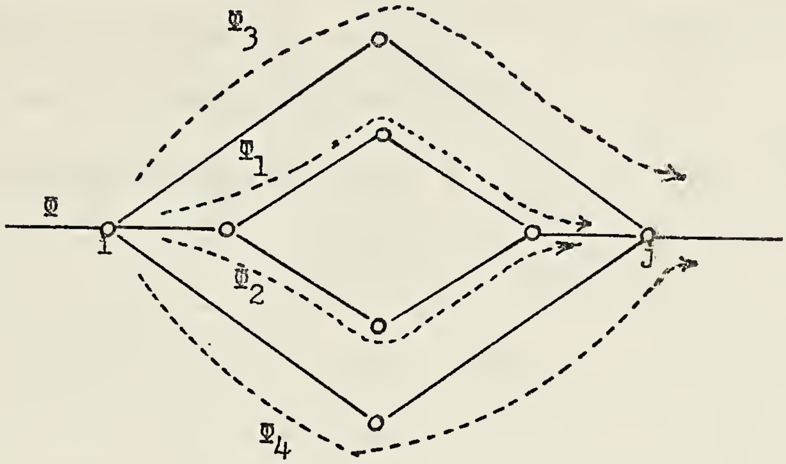

Fig. 17. A symmetric net with 4 paths

The total receiving flow at node j is equal to

$$\alpha^4(\Phi_1 + \Phi_2) + \alpha^2(\Phi_3 + \Phi_4) .$$

If the number of paths increases to $2n$, where n is a positive integer, then the total receiving flow is equal to

$$\alpha^{2n}(\Phi_1 + \Phi_2) + \alpha^{2n-2} (\Phi_3 + \Phi_4) +\ldots+ \alpha^2(\Phi_{2n-1}+\Phi_{2n}) .$$

<u>Example 11.</u> If the flows Φ_1 and Φ_2 of Fig. 16 are both equal to $\frac{\Phi}{2}$, then the receiving flow is equal to

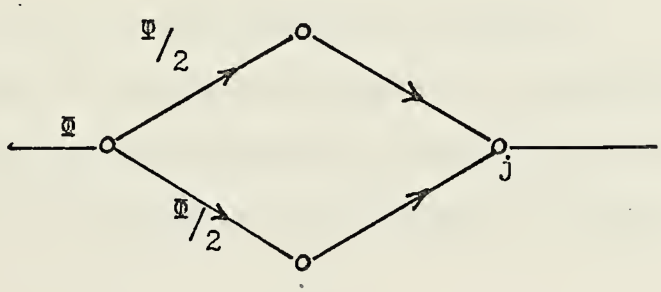

Fig. 18. Illustration for Example 11

$$\alpha^2 \Phi = (.9)^2 \Phi = .81 \Phi,$$

or 81 percent of the sending flow.

Example 12. If the flows Φ_1, Φ_2, Φ_3, and Φ_4 of Fig. 17 are all equal to $\frac{\Phi}{4}$, then the total receiving flow is

$$\alpha^4\left(\frac{\Phi}{4} + \frac{\Phi}{4}\right) + \alpha^2\left(\frac{\Phi}{4} + \frac{\Phi}{4}\right) = \alpha^4\frac{\Phi}{2} + \alpha^2\frac{\Phi}{2}$$

$$= (.9)^4 \frac{\Phi}{2} + (.9)^2 \frac{\Phi}{2}$$

$$= (.325 + .405)\,\Phi$$

$$= .73\Phi,$$

or 73 percent of the sending flow.

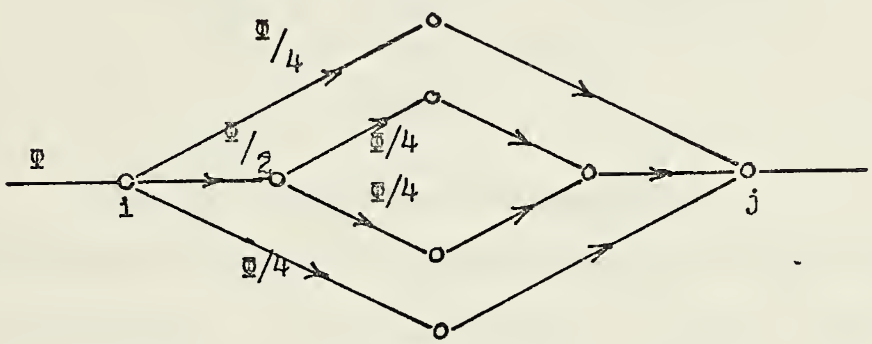

Fig. 19. Illustration for Example 12

Notice that, if the number of paths in a symmetric net increases, then the receiving flow decreases. The receiving flow is less than 70 percent of the sending flow if the number of paths is greater than 4.

C. GENERAL NETS

Example 13. Consider the net as shown in Fig. 20, in which a flow Φ is from node i to node j so that

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$

where flow Φ_1 follows path 1

flow Φ_2 follows path 2

flow Φ_3 follows path 3

flow Φ_4 follows path 4.

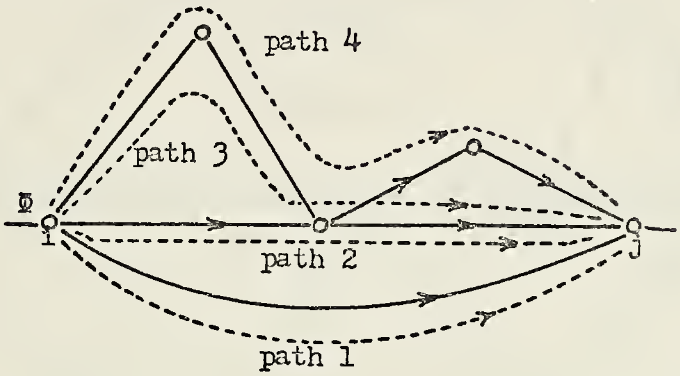

Fig. 20. Illustration for Example 13

Suppose that the capacities of all edges are large and all edges have the same efficiency. If we apply theorem 8 to path 1, path 2, path 3, and path 4 successively, then the receiving flow at node j is equal to

$$\alpha\,\Phi_1 + \alpha^2\,\Phi_2 + \alpha^3\,\Phi_3 + \alpha^4\,\Phi_4 .$$

If all the flows Φ_1, Φ_2, Φ_3, and Φ_4 are equal to $\frac{\Phi}{4}$ and all the capacities are equal to 0.9, then the total receiving flow is

$$= (.9)\frac{\Phi}{4} + (.9)^2\frac{\Phi}{4} + (.9)^3\frac{\Phi}{4} + (.9)^4\frac{\Phi}{4}$$

$$\doteq .76\,\Phi .$$

Example 14. Consider any communication net, such as the net in Fig. 21. If the capacities of all edges are large and if the efficiencies are known, then we can apply theorem 8 to this net. In this example, since path 1, path 2, and path 3 have three internal nodes, path 4 has four internal nodes, the total receiving flow at node j is equal to

$$\alpha^4 \Phi_1 + \alpha^4 \Phi_2 + \alpha^4 \Phi_3 + \alpha^5 \Phi_4 .$$

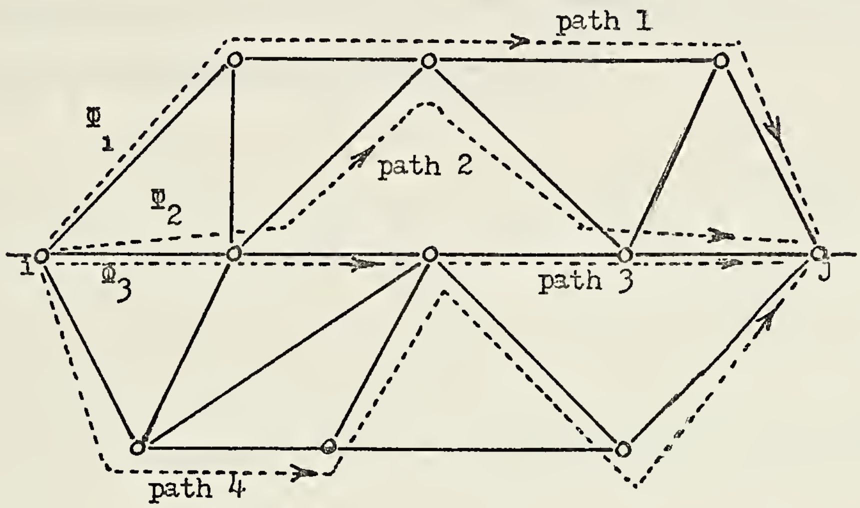

Fig. 21. A general net

V. CONCLUSION

The characteristics of the source terminal capacity and the sink terminal capacity are difficult to determine because they depend on the efficiencies, capacities, and the structure of a communication net. Some of the characteristics are discussed in this paper together with examples. Methods for obtaining sink terminal capacities in some special communication nets and the total receiving flow in a general net with equal edge capacities are presented.

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DOCUMENT CONTROL DATA - R & D

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1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE

Analysis of Terminal Capacities in Lossy Communication Nets

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; June 1973

5. AUTHOR(S) (First name, middle initial, last name)

Tran Viet Trang

6. REPORT DATE June 1973	7a. TOTAL NO. OF PAGES 34	7b. NO. OF REFS 10
8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Properties of lossy communication nets are studied in this paper. Optimum flows in general and special lossy nets are discussed. Methods for the calculations of terminal capacities in the nets are presented.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Analysis						
Terminal Capacities						
Possy Communication Nets						

Thesis 145181
T769 Trang
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Analysis of terminal
capacities in lossy com-
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